# Bending Behaviour of a Morphing Surface Connected to a Multistable Metastructure

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*Abstract*— **This paper is concerned with metastructures consisting of a configuration of bistable elements connected to a flexible non-stretching surface. These structures exploit the bistability of the elements to morph the surface into different stable bending axes. As a result of using bistable elements, the surface will maintain its shape when no external power is applied. A design of a singular, functioning, simple, bistable element is proposed which can be quickly adjusted and produced by laser cutting. In addition, an analysis of a two dimensional square configuration was carried out. A 3D morphing surface was built to verify predictions and acquire additional insight on bending behaviour. It appeared that, to have more possible stable bending axes, play and overall flexibility in the structure are desirable. The most important factor for bending behaviour of the surface turned out to be the change in the overall shape of the underlying metastructure.**

## I. INTRODUCTION

Morphing surfaces have the ability to change shape without plastic deformation. This creates potential applications, such as adaptive optics, optimizing aerodynamic efficiency by changing the shape of wings or minimizing drag of objects, at variable speeds [1].

This paper focuses on the bending behaviour of morphing surfaces, supported by a metastructure consisting of bistable elements. The working principle of the bistable elements used in this research is based on the snap-through behaviour of a beam structure [2]. The benefits of using bistability in adjustable structures are substantial, as they inherently keep their actuated position without the need for externally applied power [3]. Bistable mechanisms are therefore gaining popularity in areas such as energy harvesting, Micro-Electro-Mechanical Systems (MEMS) and metamaterials [4].

The complexity of this research lies mainly in the difficulty of predicting stability of the bending axes of the morphing metastructure. Therefore our research question is posed as follows: *Which aspects influence the bending behaviour of a morphing surface connected to a multistable metastructure?*

In this paper, the following definitions will be used: *element* meaning a single bistable mechanism, containing *flexures* which provide the translation. The elements will be linked together in *nodes* to create *unit cells*. Multiple cells will then form a *metastructure*. When a surface is attached to a metastructure, this will be called a *morphing surface*.

## II. BISTABLE ELEMENTS

Snap-through bistable elements will switch state when a critical load is applied in the direction of translation of the element. To avoid complexities in the production of bistable elements, it was decided to use a compliant mechanism that could be laser cut in one piece without further assembly. The elements used for this research are pictured in fig. 1.

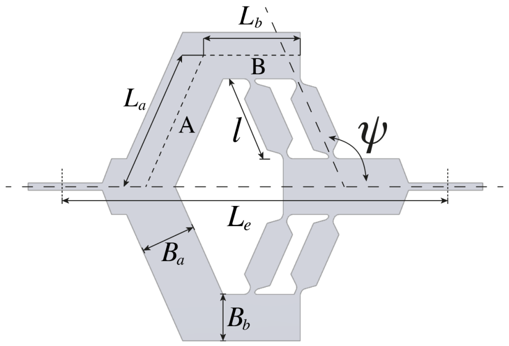


Fig. 1: *Bistable element, using hinging trusses to provide a one-directional translation. The important geometrical parameters for the snap-though behaviour of the element are shown.*

Using laser cutting as production method allowed for great customizability of the elements, but also meant the material choice narrowed down to a limited number of plastics. Of several plastics tested, High Density Polyethylene (HDPE) was eventually chosen because it created resilient flexure hinges, was cheaply available and had good laser cutting characteristics. On the downside, HDPE is known to show viscoelastic behaviour. This turned out to be an important aspect for the performance of the elements.

The snap-through behaviour of these elements is based on the elasticity of the 'Y'-shaped base, as they consist of relatively stiff hinging trusses which can switch orientation by deforming this base [5]. To establish a translation of the element without significant rotation, a double hinging truss was used on both sides.

In fig. 1 the most important geometrical parameters are shown that influence the snap-through behaviour of the bistable element. An important aspect of this behaviour is the snapping load, which is the force needed to switch the elements from one stable state to the other. The snapping load depends mainly on the angle $\psi$ of the trusses, and the dimensions (and thus stiffness) of the Y-shaped base. Using superposition, a simple analysis of the elastic deformation of beams $A$ and $B$ can be done to model the theoretical force displacement curve of an element. The result of this model is shown as the dashed red curve in fig. 2.

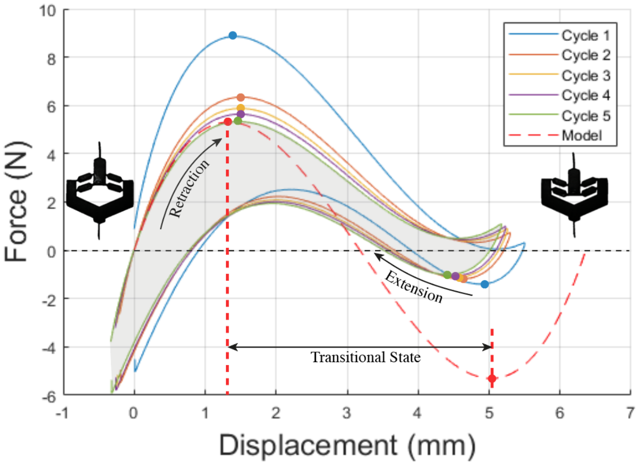


Fig. 2: *Force-displacement diagram of several consecutive compression-extension cycles of one bistable element, compared to a simple theoretical model. Positive force indicates compression, positive displacement indicates retraction of the elements. Snapping loads are indicated with dots. The shaded area shows the hysteresis for the green curve. The transitional state between extension and retraction is shown for the model curve.*

To understand more about the bistable behaviour of the elements, a tensile test was performed. In this test, elements were used that had been laser cut in extended position, like in fig. 1. The upper part of the resulting curves in fig. 2 shows the elements being compressed to their retracted state. The lower part of the curves shows the extension back to the original state. Hysteresis as a result of viscoelastic creep is clearly visible, because the extending and retracting stages of the curves are not the same. The decreasing peak forces per cycle are a result of fatigue of the material. This snapping load seems to go to a steady state value, which for the upper part of the cycle approaches the snapping load calculated in the model rather accurately.

Apart from the fact that during tensile testing the elements did not get completely extended to avoid damage, it is immediately visible that the wave shape of the measured graphs is not symmetrical. The snapping load needed to retract the elements is higher than the force needed to extend the elements back to their original position. This is a result of the resistance caused by the deformation of the flexure hinges while retracting the elements. This also explains the difference between the model and the tensile testing results, as the model does not take deformation of the hinges due to an 'original position' into account. The original laser cut position is therefore another important influence on the behaviour of the elements.

The metastructure has been designed with the elements retracted when the connected surface is flat. By extending the elements, the surface will curve away from the metastructure. This is illustrated in fig. 3. When curved, the connected morphing surface will exert a restoring force on the underlying connected framework of bistable elements, also described in other research on multistable plate structures [2]. The elements should be able to resist this force to sustain the curvature of the surface. The restoring force of the surface will increase with its curvature. For that reason, a high snapping load of the elements is especially desirable when they are extended. Consequently, it has been decided to laser cut the elements in extended position on purpose.

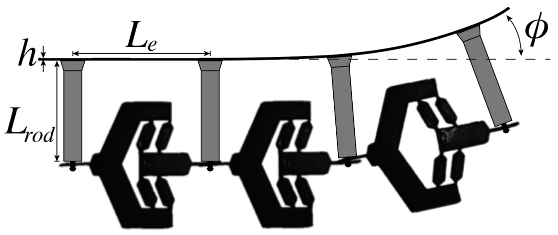


Fig. 3: *Schematic side view of elements linked together in a metastructure, with an attached surface. Rightmost element is in extended state, causing the surface to bend away from the metastructure.*

## III. METASTRUCTURE

To let the surface morph using bistable elements, the elements should be arranged in a metastructure and connected to the surface. There are many ways to arrange the bistable elements into a shape-morphing structure [6]. However, it was decided to proceed with a square geometrical configuration to have predictable behaviour.

The connection of the metastructure to the surface plays an important role in the possible bending axes [7]. A triangular truss structure between the nodes and the surface will rule out possible bending axes, as the pyramids and tetrahedrons would want to keep shape and thus restrict certain movements. Therefore a more straightforward method was chosen: straight rods fixed perpendicular to the surface on one end and the elements connected to their other end through a hinge. In order to let the entire metastructure bend out of plane, the hinges between the nodes should have two degrees of freedom: one for in-plane rotation and one for out of plane bending as shown in fig.4.

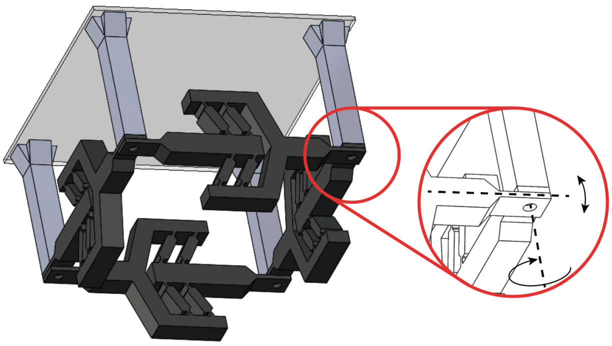
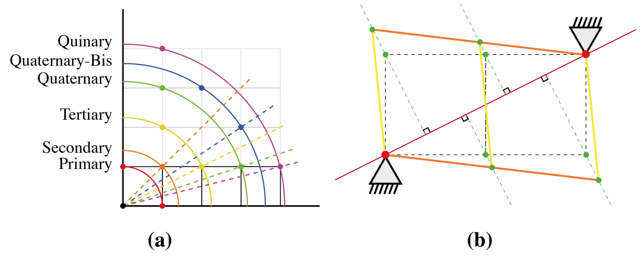


Fig. 4: *Square unit cell with rods attaching the metastructure to the surface. Zoomed in detail highlights the 2-DOF hinge, permitting both in plane rotation and out of plane hinging.*

To predict the behaviour of a morphing surface, a two dimensional geometrical analysis has been done. An appropriate terminology has been defined in order to distinguish between the different bending axes. For the two dimensional analysis, the bending axes traverse the surface through the nodes. The first adjacent intersected node represents the *primary* axis, the second represents the *secondary* axis, etcetera. A schematic extension of this terminology is shown in fig. 5a. The attached surface imposes certain constraints on the two-dimensional geometry. Assuming that the surface is only flexible in bending, but not in stretching, the nodes intersected by the bending axis will stay equidistant. Therefore, the surrounding nodes are constrained to move perpendicular to the bending axes, following their so called *working line*. This assumption is illustrated in figure 5b.



**(a)** **(b)**

Fig. 5: *Definitions of geometrical analysis. Schematic* **(a)** *illustrates the terminology of bending axes. Schematic* **(b)** *illustrating an example of the tertiary axis with its constraints: The red line shows the bending axis, while the green dotted lines illustrate the perpendicular working lines. Yellow lines indicate fully extended elements, while orange lines indicate elements in their transitional state.*

It is inherent to the properties of a bistable element that it will be in equilibrium in its extended and retracted state. For a square configuration, it is clear that both the primary and secondary axes are composed only of elements in their stable positions. For the primary axis to occur, elements perpendicular to the bending axis will have to extend. For the secondary axis, either all elements in one unit cell, or the two adjoining elements on one side of the bending axis have to extend. For both the primary and secondary axes, this is in agreement with the definition of the working lines. Therefore, it is reasonable to presume stability for both primary and secondary axes. From the tertiary axis onwards, this presumption is not as straightforward. To comply with the constraining working lines, all axes beyond the secondary axis should consist partly of elements that are in a *transitional* state. Figure 5b illustrates the transitional state of the horizontal elements in a schematic representation of the tertiary axis. As shown in figure 2, the force behaviour of the transitional state is highly nonlinear and therefore it is complicated to predict analytically if bending axes from the tertiary axis onwards will be stable. A suggested simplified way to predict stability could be based on the fact that an element in its transitional state holds potential energy. The element will naturally urge towards one of its minimal energy equilibria, i.e., the extended or retracted state. Therefore, it is expected that the bending axes beyond the secondary axis will not be stable with elements in their transitional state. This presumption is tested in an experimental way as described subsequently.

## IV. SURFACE-METASTRUCTURE INTERACTION

The bending stiffness of the surface is related to the snapping load of the connected elements, mentioned in section II. To balance the restoring force and bending stiffness of the surface, a calculation was made to predict the maximal bending stiffness of the surface with respect to the underlying metastructure. First, the bending axis at which the elements have to withstand the highest restoring force of the surface should be determined. The calculation is based on the secondary bending axis, because this axis has a higher relative extension over the working line, resulting in a stronger surface curvature. Moreover, the restoring force is not exerted axially on the elements, which requires a higher snapping load as well.

Beam theory and trigonometry were used to derive a relation between bending stiffness, rod length and extension of the elements, as shown in fig. 3. A square section of the surface was modeled. Because the element is connected diagonally under this section, it is modeled with length and width $\frac{L_e}{\sqrt{2}}$, where $L_e$ $[m]$ is the length of the element. The bending stiffness of the flexure hinges was neglected and a pure moment on the surface from the connecting rods was assumed. Because the snapping load and extension of the element are known, the maximal possible thickness $h_{max}$ $[m]$ of the surface can be calculated.

$$h_{max} \leq \sqrt[3]{\frac{6 \cdot F_e \cdot L_{rod}^2}{E \cdot \delta L}} \qquad (1)$$

Here $F_e$ $[N]$ is the maximum snapping load of the element. $\delta L$ $[m]$ is the extension of the element and $E$ $[Pa]$ indicates the Young's modulus of the surface. The width and length of the square section of the surface cancel out.

This relatively simple calculation has some limitations as the beam theory is not ideal for large deflections, and is based on a two-dimensional linear model. However, for an approximation of the maximal surface thickness, it is sufficient to determine the main parameters and the relation between them. If the thickness is chosen close to $h_{max}$, the forces on the elements will be higher and the morphing surface will be less stable.

## V. OBSERVATIONS

After creating several differently sized morphing surfaces, some non-ideal effects emerged that were not taken into account when predicting bending behaviour. Around the locations where the rods connecting the underlying metastructure are attached to the surface, local deformations of the surface take place which interrupt the continuous curvature, particularly for surfaces of low bending stiffness. For small morphing surfaces (e.g. 1×2 up to 2×3 cells), *edge effects* seemed to play an important role. The edges of the surface have more freedom than the center, because constraints of a continuing surface next to an edge are absent. The larger the created morphing surface, the smaller the influence of edges get relative to the total bending behaviour. Eventually, a 6×6 morphing surface turned out to be large enough to study the behaviour of the cells in the center with negligible edge effects.

Interestingly, for a 6×6 morphing surface, bending axes beyond the secondary axes were in fact stable, without any elements being in their transitional state. Play in the connection points and rods, as well as small local deformations of the surface, allow for nodes to deviate from their expected working lines. This enables completely retracted or extended elements to average out the required total extension of a certain row, thus allowing for more stable bending axes than just the primary and secondary.

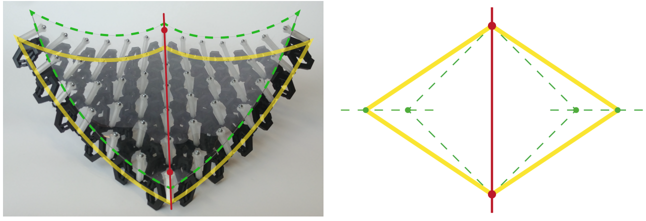

(a) (b)

Fig. 6: *Morphed surface with corresponding structural shape. Fig.* **(a)** *shows a morphed 6×6 surface for a 45° bending axis. Fig.* **(b)** *illustrates the overall shape of the underlying metastructure.*

The most important factor for bending behaviour of the surface turned out to be the *overall shape* of the underlying metastructure. Apparently, the pattern of individually extended or retracted elements is of less importance. As long as the total extension is regularly distributed over the metastructure, the angle of the bending axis can be approximated with the following relation:

$$|\tan\theta| = \frac{V}{H} \tag{2}$$

Here $\theta$ is the angle of the bending axis, $V$ is the number of extended elements in every vertical row and $H$ the number of extended elements in every horizontal row. It is interesting to note that many different configurations of extended and retracted elements can lead to the same bending axis. The number of possible configurations increases with the size of the total metastructure.

Another interesting aspect is the duality of the morphed surface. This means that, except for the primary axis, all different ratios of vertically and horizontally extended elements have a set of two different corresponding bending axes.

Furthermore, there are some situations that cannot be described correctly with relation (2). Multiple different bending axes can exist simultaneously as long as they do not intersect each other within the span of the surface, otherwise double curvature of the surface would be required. It is also possible to create a curvature gradient by increasing the number of extended elements per row, without changing the bending axes. Because the number of extended elements differs per row, relation (2) does not serve this situation.

## VI. CONCLUSION

In this paper the aspects that influence the bending behaviour of a morphing surface connected to a multistable metastructure were researched.

Even though it was impossible to evaluate all possible influences within limited research time, quite some information has been discovered. The most important findings are the following:

- Laser cutting proved to be a good production method. Although it limited the number of compatible materials and influenced snap-through behaviour, it enabled us to quickly create the elements and easily iterate the design.
- Play and overall flexibility in the surface, metastructure and connection points are actually desirable in order to obtain as many stable bending axes as possible.
- The overall shape of the metastructure is more important for the bending behaviour than the pattern of individually extended or retracted elements. This holds as long as the extended elements are regularly distributed over the metastructure.

Although the square geometry was a relatively simple configuration to analyze and work with, it could be interesting to look at other geometrical configurations and their possibilities in future research.

To be able to predict bending angles and stability of a morphing surface, it would be helpful to make a detailed model that takes the geometry of the metastructure and nonlinear bistable behaviour of the elements into account.

## VII. ACKNOWLEDGEMENTS

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