Approximation models employing design sensitivity data

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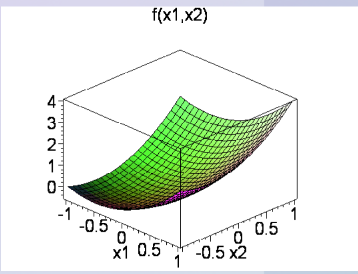
Introduction

Response surface techniques are methods to study the effect of all kinds of parameters, heat or load for instance, on a structure by performing computer simulations. Due to the high computational and economical costs of conducting these simulations, simple mathematical approximation functions, known as meta-models, are created with data from these simulations in order to replace them.
Most contemporary approximation techniques only employ the coordinates of the structures material (surface) points, which are present in the simulation data, to construct a meta-model. These coordinates are commonly named function values. However, often the derivative information that represents the slope in surface points is available inexpensively, for example in finite element analysis. This derivative information, also known as sensitivity data, combined with the most appropriate Design of Experiments (DOE), can improve the accuracy of the meta-model and cut down the costs. A DOE is a method to efficiently construct the setup of an experiment by choosing a (limited) collection of test coordinates. These test coordinates are also named design points and represent the values of the test parameters that are to be applied to perform the according test.

Problem statement

The main problem which was addressed is how to build optimal meta-models implementing design sensitivities. Here, "optimal" performance is quantified by means of the statistical model property Sum of Squared Errors (SSE). This property could be used as an accuracy measure for the global performance of the approximation function or fit as it quantifies the errors of both function and derivative data.



Meta-model f(x1,x2).

Key questions

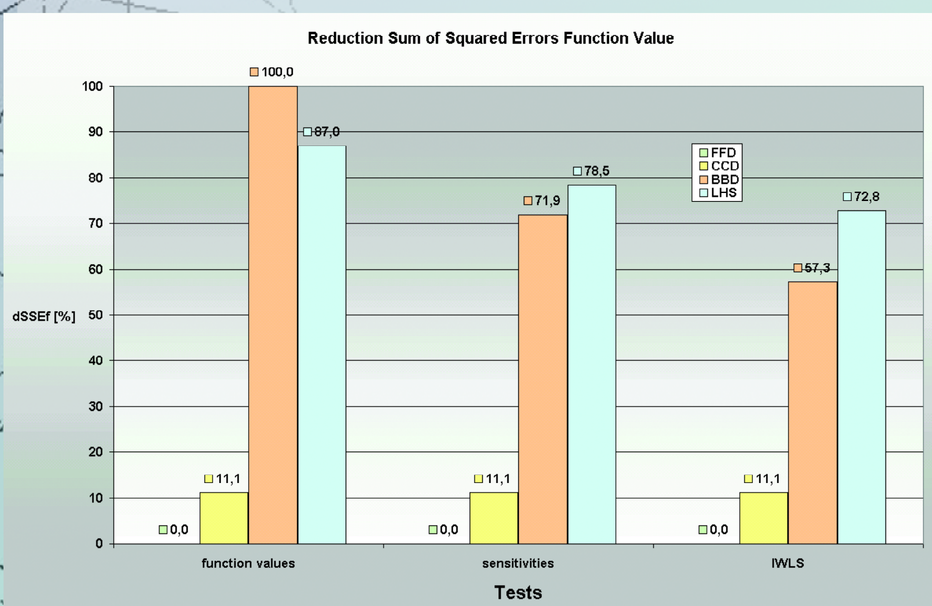
The key questions present while conducting the research were:

- Does including sensitivity data significantly increase the accuracy of approximation models?
- Which DOE produces the smallest SSE in this approximation?

Hypothesis

The SSE of function and derivative values of smooth polynomial approximation models using a Central Composite Design (CCD), Box-Behnken Design (BBD) or Latin Hybercube Sampling Design (LHS) will respectively:

- at least be 70% and 35% smaller than the SSE when using a Full Factorial Design (FFD).
- be maximally 15% higher and 40% smaller when including available design sensitivity data with the same number of design points.
- be maximally 20% higher and 45% smaller when also making use of the iterative weighted least squares (IWLS) procedure to improve weight factors for function and derivative data.



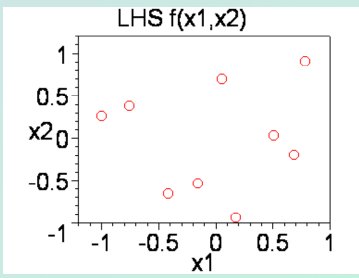
Setup of experiments

A number of test problems were used to investigate the effect of including sensitivity data and application of the four selected DOEs on the accuracy of the meta-model fits. This was achieved by performing a comparison of the reduction of the SSEs of function values and derivative data to that of a conventional FFD, which does not employ any sensitivity data, and applying the following test conditions:

- function values only
- function and sensitivity data
- function, sensitivity data and IWLS.

Results

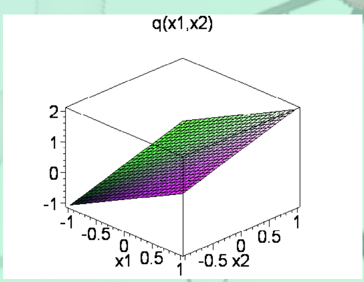
Traditional DOE theory can be applied in almost the same way to approximation functions for meta-models using sensitivity data as to those for function values only. Using a BBD or LHS design does bring about a significant error reduction, thus, complying with both hypothesis and theory. Although, the BBD seems to cause a better performance than LHS for both function values and gradients, it is actually inferior to it, as the reduction of the SSE of function values (and sometimes even derivative values) rapidly decreases by applying the IWLS procedure. LHS proves to be the best of the presented four DOEs for using function and derivative information with the tested meta-model approximation functions.



Latin Hypercube Sampling Design for approximation of meta-model f(x1,x2).

Conclusion and Recommendations

The main conclusion that can be drawn is that derivative data can indeed be included efficiently to approximate meta-models. By employing derivatives and applying the IWLS procedure to improve weight factors for function and derivative data a considerable global improvement is gained over approximations based on function values only. The Latin Hypercube Sampling Design proved to be the best DOE. Future studies could investigate the influence of characteristic parameters of engineering problems, for example, large numbers of design variables.



Approximation function q(x1,x2) of meta-model f(x1,x2).